A Proof of the Riemann Hypothesis

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Abstract:

A proof of the Riemann Hypothesis is proposed in six lemmas, where five of the six are proven using elementary arithmetic. It is shown that by applying all the zeros of the zeta function to a ratio, having an infinite number of numerators and divisors equal to the same value, the modulus of a variable z used to calculate the ratio are all equal to the square root of one divided by fourteen for all the zeros of zeta of s, trivial or non-trivial. Using the common modulus, it is shown that the value of the ratio for all the non-trivial zeros is a fixed constant, whereby allowing one to calculate the only possible positive Real part of s for the non-trivial zeros. Such proof suggests that the greatest common multiple and lowest common denominator of this ratio for all the zeros of zeta of s lie in the non-trivial zeros with a fixed Real part one half.

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Lemma 1:

Let an arbitrary sum *s* (not necessarily a vector)

$$s = R + I \tag{1}$$

And a ratio with an arbitrary delta ≠ 0 or delta ≠ nu: T_R

$$T_R = \frac{\nu}{\delta} \tag{2}$$

Such that,

$$T_R = \frac{(2R + 1)^{-1} - 4}{4} \tag{3}$$

Given the function phi of *x*

$$\phi\,(x) = 1 + \frac{\nu^1}{\delta} + \frac{\nu^2}{\delta} + \frac{\nu^3}{\delta} + \frac{\nu^4}{\delta} + \frac{\nu^5}{\delta} + \ldots \tag{4}$$

Where phi of *x* equals the sum over all integers greater than minus one of nu to the *x* divided by delta,

And given the function kappa of *x*,

$$\kappa\,(x) = 1 + \nu^2 + \nu^4 + \nu^5 + \nu^6 + \nu^7 + \nu^8 + \nu^9 + \nu^{10} + \ldots \tag{5}$$

Where kappa of *x* equals the sum over all integers greater than minus one except one and three of nu to the *x*,

And given the function lambda of nu,

$$\lambda\,(\nu) = \prod_p (1 - \nu^p)^{-1} \tag{6}$$

Where lambda of nu equals the product over all primes of one minus nu to the *p*, to the minus one,

One gets

The Lemma:

$$\lambda (\nu) = \frac{\phi (x) - T_R}{\kappa (x)} \tag{7}$$

Proof of Lemma 1:

Given phi of *x*, subtract both sides of equation (4) by T_R (or nu to the one divided by delta, which equals nu divided by delta). This gives

$$\phi (x) - T_R = 1 + \frac{\nu^2}{\delta} + \frac{\nu^3}{\delta} + \frac{\nu^4}{\delta} + \frac{\nu^5}{\delta} + \frac{\nu^6}{\delta} + \ldots \tag{8}$$

Multiply both sides by nu to the first prime divided by delta. This gives

$$\frac{\nu^2}{\delta} (\phi (x) - T_R) = \frac{\nu^2}{\delta} + \frac{\nu^4}{\delta} + \frac{\nu^5}{\delta} + \frac{\nu^6}{\delta} + \frac{\nu^7}{\delta} + \ldots \tag{9}$$

Multiply both sides by delta. This gives

$$\nu^2 (\phi (x) - T_R) = \nu^2 + \nu^4 + \nu^5 + \nu^6 + \nu^7 + \nu^8 + \nu^9 + \nu^{10} + \ldots \tag{10}$$

Subtract the second of the expressions by the first. This gives

$$(1 - \nu^2) (\phi (x) - T_R) = 1 + \nu^2 + \nu^3 + \nu^4 + \nu^5 + \nu^6 + \nu^7 + \nu^8 + \ldots \tag{11}$$

Invert both sides. By means of logarithmic rules, this gives

$$(1 - \nu^2)^{-1} (\phi (x) - T_R)^{-1} = (1 + \nu^2 + \nu^3 + \nu^4 + \nu^5 + \nu^6 + \ldots)^{-1} \tag{12}$$

Multiply both sides by one minus nu to the next consecutive prime, to the minus one. This gives

$$(1 - \nu^3)^{-1} (1 - \nu^2)^{-1} (\phi(x) - T_R)^{-1} = (1 + \nu^2 + \nu^4 + \nu^5 + \ldots)^{-1} \tag{13}$$

Where the inverse of the infinite series converges to the inverse of kappa of *x* from equation (5).

Multiply both sides of equation (13) by one minus nu to the consecutive prime to the minus one. This gives

$$(1 - \nu^5)^{-1} (1 - \nu^3)^{-1} (1 - \nu^2)^{-1} (\phi(x) - T_R)^{-1} = (1 + \nu^2 + \nu^4 + \nu^5 + \ldots)^{-1} \tag{14}$$

Where the inverse of the infinite series on the right remains equal to the inverse of kappa of *x*.

By repeating the steps above for all consecutive primes in turn, the right side of the equation remains converged at the inverse of kappa of *x*.

$$\ldots (1 - \nu^7)^{-1} (1 - \nu^5)^{-1} (1 - \nu^3)^{-1} (1 - \nu^2)^{-1} (\phi(x) - T_R)^{-1} = \frac{1}{\kappa(x)} \tag{15}$$

Dividing each side of the expression repeatedly by each of the parentheses in turn to the last before the equals sign, one gets

$$\frac{1}{\phi(x) - T_R} = \frac{1}{\kappa(x)} (1 - \nu^2)(1 - \nu^3)(1 - \nu^5)(1 - \nu^7)(1 - \nu^{11}) \ldots \tag{16}$$

Invert and divide both sides by kappa of *x*. This gives

$$\frac{\phi(x) - T_R}{\kappa(x)} = (1 - \nu^2)^{-1} (1 - \nu^3)^{-1} (1 - \nu^5)^{-1} (1 - \nu^7)^{-1} (1 - \nu^{11})^{-1} \ldots \tag{17}$$

Where the product on the right hand side is that of lambda of nu, as proof for the lemma of (7) completed below.

$$\frac{\phi(x) - T_R}{\kappa(x)} = \lambda(\nu) \tag{18}$$

Lemma 2:

The lemma:

$$\zeta (s) = \prod_{p} (1 - p^{p \log\_e (\nu) \div \log\_e (p)})^{-1} \tag{19}$$

Where zeta of *s* equals the product over all primes of one minus *p* to the *p* times the natural logarithm of nu from equations (2-18) divided by the natural logarithm of *p*, to the minus one.

Proof of Lemma 2:

Given the Riemann zeta function

$$\zeta (s) = \prod_{p} (1 - p^{-s})^{-1} \tag{20}$$

Where zeta of *s* equals the product over all primes of one minus *p* to the minus *s*, to the minus one, which is the so-called Riemann Zeta Function,

And given the product from equation (6) and an arbitrary *s* from (1), one can derive a value nu for the zeta function by letting

$$\frac{1}{1 - p^{-s}} = \frac{1}{1 - \nu^{p}} \tag{21}$$

Where the left side is from the Riemann zeta function in (20) and the right side is from the lambda function in (6).

Invert both sides. One gets

$$1 - p^{-s} = 1 - \nu^{p} \tag{22}$$

Subtract one from both sides. This gives

$$p^{-s} = \nu^{p} \tag{23}$$

Give the natural logarithm of both sides, and by means of logarithmic rules bring out minus *s* and *p* from the parentheses on both sides. One gets

$$-s \log_e (p) = p \log_e (\nu) \tag{24}$$

Divide both sides by *p*. One gets

$$\frac{-s \log_e (p)}{p} = \log_e (\nu) \tag{25}$$

Give the exponent of both sides. One gets

$$e^{-s \log\_e (p) \div p} = \nu \tag{26}$$

Thus, for any given *s* of the zeta function, one can solve for a value nu to be applied as the numerator of T_R for arguments in phi of *x*, kappa of *x* and lambda of nu. One can then rearrange equation (24) to apply the numerator of T_R to the zeta function in place of *s* in order to fulfill the proof for Lemma 2.

Using equation (24), divide both sides by the minus natural logarithm of *p*. One gets

$$s = \frac{p \log_e (\nu)}{-\log_e (p)} \tag{27}$$

Where one gets an arbitrary nu as an argument for the zeta function.

Let the primes be raised on both sides of the equals sign, subtract by one and raise both sides to the power of minus one. One gets

$$(1 - p^{-s})^{-1} = (1 - p^{p \log\_e (\nu) \div -\log\_e (p)})^{-1} \tag{28}$$

Where the proof for Lemma 2 in (28) is completed by continuing to take the product over all primes of one minus *p* to the *p* natural logarithm of nu divided by the minus natural logarithm of *p*, to the minus one.

Lemma 3:

The lemma:

The numerators of T_R for all the non-trivial zeros of the zeta function equal one.

Proof of Lemma 3:

Using the Riemann zeta function from equation (20), express the product expanded. This gives

$$\zeta (s) = (1 - 2^{-s})^{-1} (1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots \qquad (29)$$

Take the natural logarithm of both sides. One gets

$$\log_e (\zeta (s)) = \log_e ((1 - 2^{-s})^{-1} (1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} \ldots) \qquad (30)$$

By means of logarithm rules express the right side as a series of logarithms. This gives

$$\log_e (\zeta (s)) = \log_e ((1 - 2^{-s})^{-1}) + \log_e ((1 - 3^{-s})^{-1}) + \log_e ((1 - 5^{-s})^{-1}) + \ldots \qquad (31)$$

Subtract both sides by the natural logarithm of one minus two to the minus *s*, to the minus one. One gets

$$\log_e (\zeta (s)) - \log_e ((1 - 2^{-s})^{-1}) = \log_e ((1 - 3^{-s})^{-1}) + \log_e ((1 - 5^{-s})^{-1}) + \ldots \qquad (32)$$

By means of logarithmic rules express the left side as the natural logarithm of a ratio. Because the denominator of such ratio is raised to a minus one, complete by reducing to the log of a product. This gives

$$\log_e (\zeta (s) (1 - 2^{-s})) = \log_e ((1 - 3^{-s})^{-1}) + \log_e ((1 - 5^{-s})^{-1}) + \ldots \qquad (33)$$

By means of logarithm rules, express the right side as the logarithm of a product and give the exponent of both sides. One gets

$$\zeta (s) (1 - 2^{-s}) = (1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots \qquad (34)$$

Invert both sides. One gets

$$\zeta (s)^{-1} (1 - 2^{-s})^{-1} = (1 - 3^{-s}) (1 - 5^{-s}) (1 - 7^{-s}) (1 - 11^{-s}) \ldots \qquad (35)$$

Multiply both sides by zeta of *s*. One gets

$$(1 - 2^{-s})^{-1} = \zeta (s) (1 - 3^{-s}) (1 - 5^{-s}) (1 - 7^{-s}) (1 - 11^{-s}) \ldots \quad (36)$$

Give the natural logarithm of both sides and by means of logarithmic rules, bring out the power of minus one from the left side and express the right side as a series of logarithms. One gets

$$-1 \log_e (1 - 2^{-s}) = \log_e (\zeta (s)) + \log_e (1 - 3^{-s}) + \log_e (1 - 5^{-s}) + \ldots \quad (37)$$

Divide both sides by minus one. One gets

$$\log_e (1 - 2^{-s}) = -\log_e (\zeta (s)) - \log_e (1 - 3^{-s}) - \log_e (1 - 5^{-s}) - \ldots \quad (38)$$

By means of logarithmic rules, express the right side as a natural logarithm of a ratio. One gets

$$\log_e (1 - 2^{-s}) = -\log_e (\zeta (s) \div (1 - 3^{-s}) \div (1 - 5^{-s}) \div (1 - 7^{-s}) \div \ldots) \quad (39)$$

Rearrange everything to the right of zeta of *s* by means of inversion to the following:

$$\log_e (1 - 2^{-s}) = -\log_e (\zeta (s) \div (1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} \ldots) \quad (40)$$

Express the right side as a multiple of minus one, and by means of logarithmic rules invert the value of the logarithm on the right and give the exponent of both sides. One gets

$$\frac{1}{1 - 2^{-s}} = \frac{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots}{\zeta (s)} \quad (41)$$

Invert both sides again. This gives

$$1 - 2^{-s} = \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots} \quad (42)$$

Add minus one to both sides and then multiply both by minus one. One gets

$$2^{-s} = 1 - \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots} \quad (43)$$

Take the natural logarithm of both sides, and by means of logarithmic rules bring out minus *s* from the left side. One gets

$$-s \log_e (2) = \log_e (1 - \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots}) \qquad (44)$$

By means of equation (27), substitute minus *s* for the first nu of zeta of *s* on the left. One gets

$$\frac{\log_e (2)\ 2 \log_e (\nu_1)}{\log_e (2)} = \log_e (1 - \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} \ldots}) \qquad (45)$$

The natural logarithm of two cancels from the left side. One gets

$$2 \log_e (\nu_1) = \log_e (1 - \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots}) \qquad (46)$$

Divide both sides by 2. One gets

$$\log_e (\nu_1) = \frac{1}{2} \log_e (1 - \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots}) \qquad (47)$$

Give the exponent of both sides. One gets

$$\nu_1 = \exp (\frac{1}{2} \log_e (1 - \frac{\zeta (s)}{(1 - 3^{-s})^{-1} (1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots})) \qquad (48)$$

Given zeta of *s* equal to zero with a positive Real part of *s*, by order of operations the logarithm of the mixed number on the right reduces to one. One gets

$$\nu_1 = e^{\log\_e (1) \div 2} \qquad (49)$$

Where the base of natural logarithms to the natural logarithm of one divided by two reduces to one, as the natural logarithm of one equals zero and *e* to the zero equals one. One gets

$$\nu_1 = 1 \qquad (50)$$

By repeating the above for the next value of nu, one gets

$$\nu_2 = \exp \left(\frac{1}{2 \times 3} \log_e \left(1 - \frac{\zeta (s)}{(1 - 5^{-s})^{-1} (1 - 7^{-s})^{-1} (1 - 11^{-s})^{-1} \ldots}\right)\right) \tag{51}$$

Where the second nu from zeta of *s* also equals one.

Repeat the same for all values of nu from zeta of *s*. This gives

$$\nu_\infty = \exp \left(\frac{1}{2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times 23 \ldots} \log_e (1)\right) \tag{52}$$

Where the natural logarithm of one remains, which is equal to zero in the exponent, as a multiple of the inverse of the product of all primes. And because any number multiplied by zero equals zero, all the nu's remain constant when zeta of *s* equals zero and the Real part of *s* is positive, which is equal to one.

Equations (29 – 52), as well as the process involved can be verified for illustrative purposes by applying the numerators of T_R for the known non-trivial zeros to equation below.

Using Equation (26) and equation (53) below, the definition of e^r for any number r

$$e^r = 1 + r + \frac{r^2}{1 \times 2} + \frac{r^3}{1 \times 2 \times 3} + \frac{r^4}{1 \times 2 \times 3 \times 4} + \ldots \tag{53}$$

Where

$$r = \frac{-s \log_e (p)}{p} \tag{54}$$

One can calculate the numerator of T_R for all the known non-trivial zeros of zeta of *s*, where the Imaginary part of *s* for all prime numbers *p* in equation (54) closes in on zero in an oscillatory manner and the Real part closes in on one. The larger the prime number, the faster equation (53) converges; the smaller the prime number, the slower it converges. But just as the proof for lemma shows for all the non-trivial zeros of zeta of *s*, equation (53) can confirm for at least all the known non-trivial zeros.

Lemma 4:

Let the denominator from equation (2) be defined as the function delta of *z*. Given delta of *z*

$$\delta (z) = \frac{4 (z + 1)}{(-4z)^{-1} - 4} \tag{55}$$

The lemma:

The limit for the modulus of all trivial zero *z's* of the zeta function equals the square root of one divided by fourteen.

Proof of Lemma 4:

Multiply both sides of equation (55) by the denominator of the right hand side. This gives

$$\frac{\delta (z)}{-4z} - 4\,\delta (z) = 4 (z + 1) \tag{56}$$

Divide both sides by four. One gets

$$\frac{\delta (z)}{-16z} - \delta (z) = z + 1 \tag{57}$$

Multiply both sides by *z*. This gives

$$\frac{\delta (z)}{-16} - z\,\delta (z) = z^2 + z \tag{58}$$

Add *z* times delta of *z* to both sides and also add delta of *z* divided by sixteen to both sides. One gets

$$0 = z^2 + z + z\,\delta (z) + \frac{\delta (z)}{16} \tag{59}$$

By means of the Quadratic Equation, where

$a = 1$
$b = 1 + \delta (z)$
$c = \delta (z) \div 16$

One gets

$$z = \frac{-(1 + \delta (z)) \pm \sqrt{(1 + \delta (z))^2 + (\delta (z) \div 4)}}{2} \tag{60}$$

Begin by solving for the *z*'s of the first trivial zero of zeta of *s,* which is equal to minus two. Using equation (26), solve for the first value of the numerator from equation (2), corresponding to the first prime number. This gives

$$\nu_1 = e^{2 \log\_e (2) \div 2 = 2} = 2 \tag{61}$$

Then solve for the next nu where *s* equals minus two, using the next prime number. One gets,

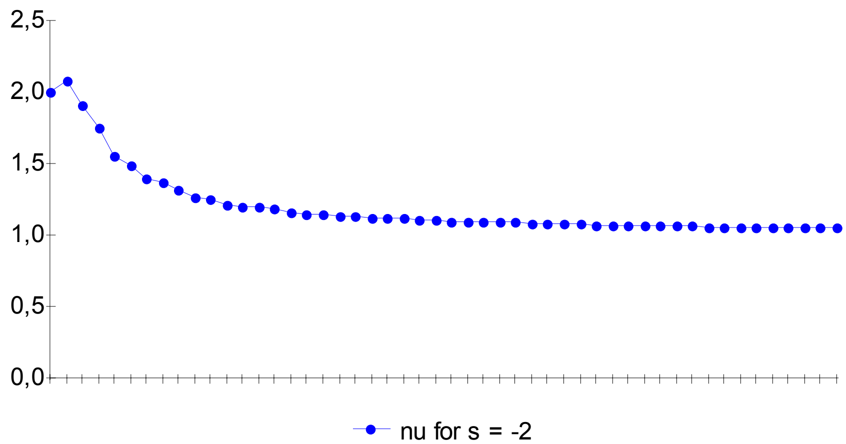
$$\nu^2 = e^{2 \ln (3) \div 3} = 2.080083\ldots \tag{62}$$

And then continue with the next prime number. This gives

$$\nu^3 = e^{2 \ln (5) \div 5} = 1.903653\ldots \tag{63}$$

Continuing through all the primes, the function converges quickly on one.

FIGURE 1 The function ν for s = -2



Next, invert both sides of equation (3) and express T_R as nu divided by delta. One gets,

$$\frac{\delta}{\nu} = \frac{4}{(2R + 1)^{-1} - 4} \tag{64}$$

Multiply both sides by nu and solve for delta for the Real part of *s* equal to minus two and the first nu equal to two. One gets,

$$\delta_1 = \frac{4 \text{ x } 2}{(2 \text{ x } -2 + 1)^{-1} - 4} = -1.846153\ldots \tag{65}$$

Then using equation (60), solve for the values of *z* for the Real part of *s* equal to minus two. One gets the first following values,

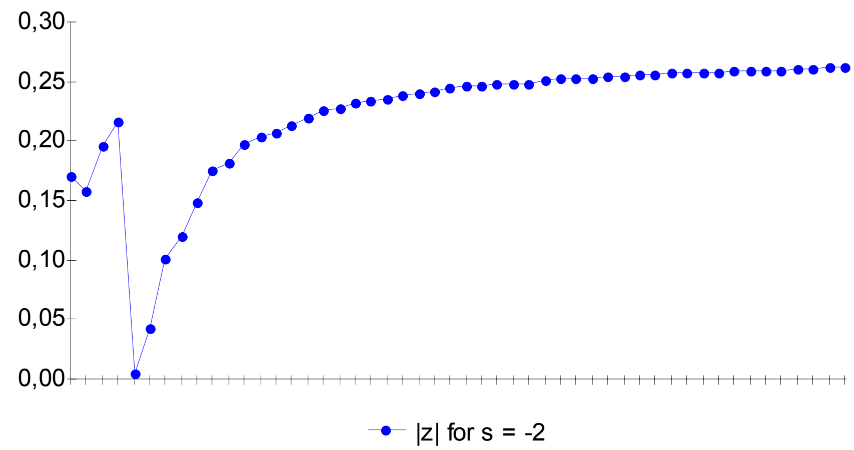
TABLE 1 The z Function for s = -2

z	+	-
z_1	0.675286...	0.170867...
z_2	0.762744...	0.157332...
z_3	0.561691...	0.195527...
z_4	0.392608...i	0.216904...i
z_5	0.422390...i	0.005129...i
z_6	0.411638...i	-0.041974...i
z_7	0.388546...i	-0.100307...i

By repeating this process for all the trivial zeros, one gets the same case where the number of Real values of *z* increase as the absolute value of *s* increases, but where all the values eventually become

Imaginary. This functions converges on the Imaginary square root of one divided by fourteen. And since the Imaginary part *s* equal to minus two has a Real part zero, the modulus for *z* for *s* equal to minus two converges on the Real square root of one divided by fourteen.

FIGURE 2 The modulus of z for s = -2



Repeat in the same for the consecutive trivial zeros and one gets that the modulus of *z* for all the trivial zeros converges in the same onto the square root of one divided by fourteen.

Lemma 5:

Given the function

$$M_z(x) = |z_1| \div \{|z_2| \div \{|z_3| \div \{|z_4| \div \ldots \quad (66)$$

$$+ |z_3| \div \{|z_4| \div \{|z_5| \div \{|z_6| \div \ldots$$

$$+ |z_4| \div \{|z_5| \div \{|z_6| \div \{|z_7| \div \ldots + \ldots\}\}\}$$

$$+ |z_5| \div \{|z_6| \div \{|z_7| \div \{|z_8| \div \ldots + \ldots\}\}\}\}$$

$$+ |z_4| \div \{|z_5| \div \{|z_6| \div \{|z_7| \div \ldots$$

$$+ |z_5| \div \{|z_6| \div \{|z_7| \div \{|z_8| \div \ldots + \ldots\}\}\}$$

$$+ |z_6| \div \{|z_7| \div \{|z_8| \div \{|z_9| \div \ldots + \ldots \}\}\}\}\}\}$$

$$+ |z_3| \div \{|z_4| \div \{|z_5| \div \{|z_6| \div \ldots$$

$$+ |z_4| \div \{|z_5| \div \{|z_6| \div \{|z_7| \div \ldots$$

$$+ |z_5| \div \{|z_6| \div \{|z_7| \div \{|z_8| \div \ldots + \ldots \}\}\}\}$$

$$+ |z_6| \div \{|z_7| \div \{|z_8| \div \{|z_9| \div \ldots + \ldots \}\}\}\}\}\}\}\}$$

$$+ |z_5| \div \{|z_6| \div \{|z_7| \div \{|z_8| \div \ldots$$

$$+ |z_6| \div \{|z_7| \div \{|z_8| \div \{|z_9| \div \ldots + \ldots \}\}\}\}$$

$$+ |z_7| \div \{|z_8| \div \{|z_9| \div \{|z_{10}| \div \ldots + \ldots \}\}\}\}\}\}\}\}\}\}$$

Which can be illustrated as Figure 3,

FIGURE 3 The function M of z, of x

$$\cfrac{|z_1|}{\cfrac{|z_2|}{\cfrac{|z_3|}{\cfrac{|z_4|}{\downarrow \atop \infty} + \cfrac{|z_5|}{\downarrow \atop \infty}} + \cfrac{|z_4|}{\cfrac{|z_5|}{\downarrow \atop \infty} + \cfrac{|z_6|}{\downarrow \atop \infty}}} + \cfrac{|z_3|}{\cfrac{|z_5|}{\cfrac{|z_6|}{\downarrow \atop \infty} + \cfrac{|z_7|}{\downarrow \atop \infty}} + \cfrac{|z_6|}{\cfrac{|z_7|}{\downarrow \atop \infty} + \cfrac{|z_8|}{\downarrow \atop \infty}}}}$$

The lemma:

By dividing out each modulus of *z* from the denominator function in equation (55), the limit of *M* of *z*, of *x* is one half for all the zeros of the zeta function, trivial and non-trivial.

Proof of Lemma 5:

Apply all the first values of *z* for all the trivial zeros, where each of the first numerators of T_R for the trivial zeros begin as

TABLE 2 The Function ν_1 (s)

s	ν_1
LCD	Reserved
-2	2
-4	4
-6	8
-8	16
-10	32
-12	64

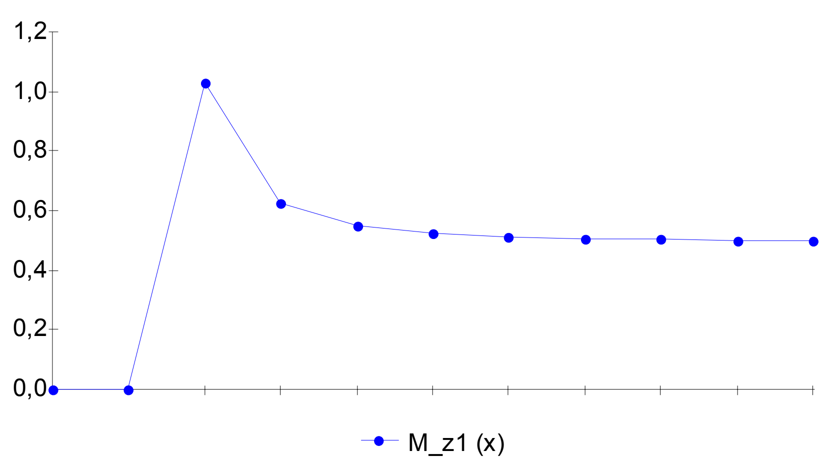
Into *M* of *z,* of *x* from equation (66), beginning with the first *z* of *s* equal to minus two equal to the second value of *M* of *z,* of *x*, reserving the first value for the lowest common denominator. This can be done in the following manner:

$$M_{z1}(2) = \frac{|z_2|}{|z_3| + |z_4|} = \frac{0.1708...}{0.0869... + 0.0724...} \tag{67}$$

$$M_{z1}(3) = \frac{|z_3|}{|z_4| + |z_5|} = \frac{0.0869...}{0.0724... + 0.0670...} \tag{68}$$

And so on toward *x* equal to infinity.

FIGURE 4 The function M of z one, of x



The values for *M* of *z* one, of *x* begin as

TABLE 3 The Function M_{z1} (x)

x	M_z1
LCD	Reserved
2	1.071869…
3	0.623625…
4	0.549854…
5	0.522781…
6	0.510918…
7	0.505345…

This function rapidly converges on one half.

$$\lim_{x \to \infty} M_{z1}(x) = \tfrac{1}{2} \tag{69}$$

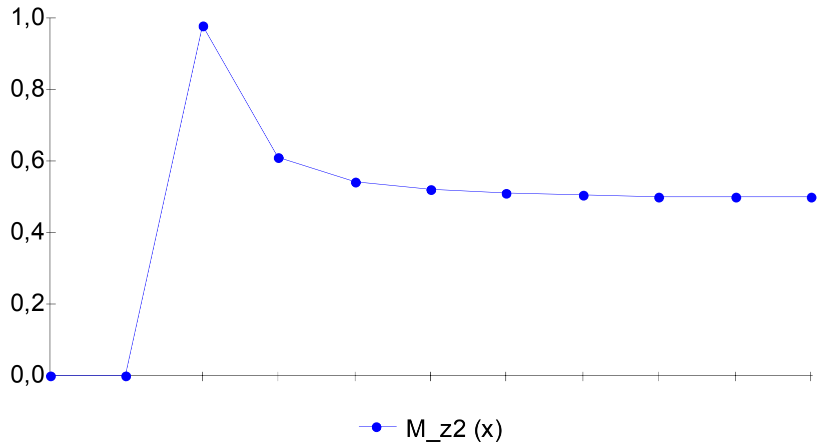
Next place the second values of *z* for all the trivial zeros, where each of the second numerators of T_R for the trivial zeros begin as

TABLE 4 The Function ν_2 (s)

s	ν_2
LCD	Reserved
-2	2.080084…
-4	4.326749…
-6	9
-8	18.720754…
-10	38.940738…
-12	81

Into *M* of *z* of *x* from equation (66), beginning with the second *z* of *s* equal to minus two equal to the second value of *M* of *z,* of *x*, again reserving the first value for the lowest common denominator.

FIGURE 5 The function M of z two, of x



The values of *M* of *z* two, of *x* begin as

TABLE 5 The Function M_{z2} (x)

x	M_z2
LCD	Reserved
2	1.011308…
3	0.613885…
4	0.544794…
5	0.519836…
6	0.509184…
7	0.504335…

This function also converges rapidly on one half.

$$\lim_{x \to \infty} M_{z2}(x) = \frac{1}{2} \qquad (70)$$

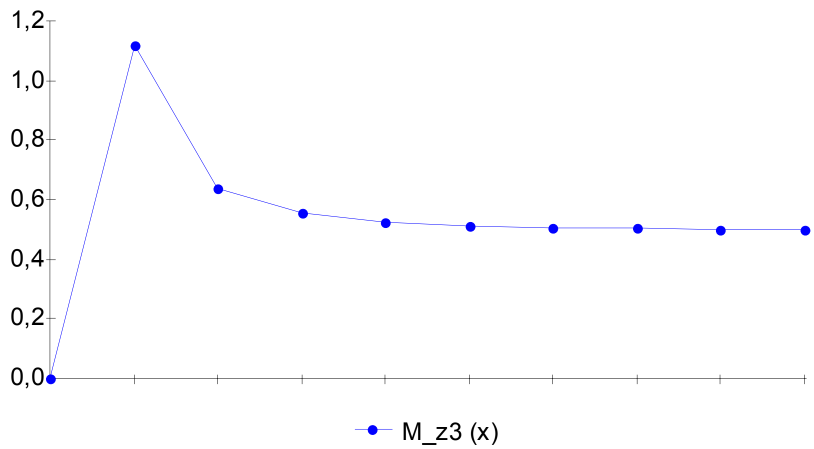
Next, place all third values of *z* for all the Trivial Zeros, where each of the third numerators of T_R for the trivial zeros begin as

TABLE 6 The Function ν_3 (s)

s	ν_3
LCD	Reserved
-2	1.903654…
-4	3.623898…
-6	6.898648…
-8	13.132639…
-10	25
-12	47.591348…

Into *M* of *z,* of *x* from equation (66), beginning with the third *z* of *s* equal to minus two equal to the second value of *M* of *z*, of *x*, again reserving the first value for the lowest common denominator.

FIGURE 6 The function M of z three, of x



The values of *M* of *z* three, of *x* begin as

TABLE 7 The Function M_{z3} (x)

x	M_z3
LCD	Reserved
2	1.183932…
3	0.637383…
4	0.557047…
5	0.527086…
6	0.513545…
7	0.506937…

This function also converges rapidly on one half.

$$\lim_{x \to \infty} M_{z3} (x) = ½ \qquad (71)$$

By repeatedly placing all the following values of *z* for the Trivial Zeros into *M* of *z,* of *x* in the same manner. One gets,

$$\lim_{x \to \infty} M_{zx \to \infty} (x) = ½ \qquad (72)$$

By fulfilling this proceedure, one not only gets the result of equation (72), but also a method for determining the prime factors of the absolute value of *s*, as the value of the numerator of T_R is only an Integer when the corresponding prime is a factor of the absolute value of *s*. Below are a few tables to demonstrate this.

TABLE 8 The function ν (x) for s = -18

x	**primes**	**ν (x)**
1	2	512
2	3	729
3	5	323.315...
4	7	148.973...
5	11	50.593...
6	13	34.864...

Where the prime factors of the absolute value of *s*, as well as five hundred twelve and seven hundred twenty nine are two and three.

$$18 = 2 \times 3 \times 3$$
$$512 = 2^{(3 \times 3)}$$
$$729 = 3^{(3 \times 2)}$$

TABLE 9 The function ν (x) for s = -28

x	**primes**	**ν (x)**
1	2	16384
2	3	28387.798...
3	5	8207.899...
4	7	2401
5	11	447.525...
6	13	250.762...

Where the prime factors of the absolute value of *s*, as well as sixteen thousand three hundred eighty four and two thousand four hundred one are two and seven.

$$28 = 2 \times 2 \times 7$$
$$16384 = 2^{(2 \times 7)}$$
$$2401 = 7^{(2 \times 2)}$$

TABLE 10 The function ν (x) for s = -40

x	**primes**	**ν (x)**
1	2	1048576
2	3	2299411.661...
3	5	390625
4	7	67473.254...
5	11	6121.847...
6	13	2676.196...

Where the prime factors of the absolute value of *s*, as well as one million forty eight thousand five hundred seventy six and three hundred ninety thousand six hundred twnety five are two and five.

40 $= 2 \times 2 \times 2 \times 5$
1048576 $= 2^{(2 \times 2 \times 5)}$
390625 $= 5^{(2 \times 2 \times 2)}$

This can be extended to any negative value of *s* to determine the prime factors of the absolute value of *s*, even if minus *s* is not a zero of the zeta function, as shown in FIGURE 11.

TABLE 11 The function ν (x) for s = -35

x	**primes**	**ν (x)**
1	2	185363.800...
2	3	368480.609...
3	5	78125
4	7	16807
5	11	2048.364...
6	13	997.879...

Where the prime factors of the absolute value of *s*, as well as seventy eight thousand one hundred twenty five and sixteen thousand eight hundred seven are five and seven.

35 $= 5 \times 7$
78125 $= 5^7$
16807 $= 7^5$

This also provides the means to determine if a number is prime or not—for when *s* is a minus prime number, the only Integer in nu of *x* is the absolute value of *s*, and the corresponding prime to that Integer is also the absolute value of *s*.

TABLE 12 The function ν (x) for s = -11

x	**primes**	**ν (x)**
1	2	45.254...
2	3	56.162...
3	5	34.493...
4	7	21.281...
5	11	11
6	13	8.761...

Where the absolute value of *s* is prime.

$11 = \nu\ (5),\ p_5$

The significance of this for the purposes of this paper is that it gives evidence that all nu of *x*, even those not Integers, are multiples to some power, of the Real part of *s*. Such allows the division of each modulus for all *z*, as done in equation (72), to determine the lowest common denominator of all the common values of a function *s* is applied to; in this case, all the zeros of the zeta function.

Lemma 6:

Given the function *H* of T_R

$$H(T_R) = \sum_{x=1} T_R^{x} \tag{73}$$

Where T_R is the ratio from equation (2) and where *H* of T_R converges when the absolute value of nu equals one, as in the case of all the non-trivial zeros of the Zeta Function by proof of Lemma 3,

And given a function *P* of T_R, dependent upon *H* of T_R, which also converges when the absolute value of nu equals one

$$P(T_R) = \frac{(H(T_R) + 1)(T_R + 1)}{s} \tag{74}$$

And given the function *K* of T_R, dependent upon *P* of T_R

$$K(T_R) = \frac{T_R + T_R \bmod P(T_R)}{P(T_R)} \tag{75}$$

When *s* from equation (74) is applied as an argument to the zeta function, one gets

The Lemma:

The Real part of *s* for all the non-trivial zeros of zeta of *s* becomes the lowest common denominator for *P* of T_R , as well as the limit of *M* of *z*, of *x*,

Resulting in

$$P(T_R) = \frac{-1}{K(T_R) - s} \tag{76}$$

Which provides the means to express all the zeros of zeta of *s*, trivial or not, in terms of the modulus of *z* from equation (55), so long as the minimum and maximum limits of the following are known in closed form as

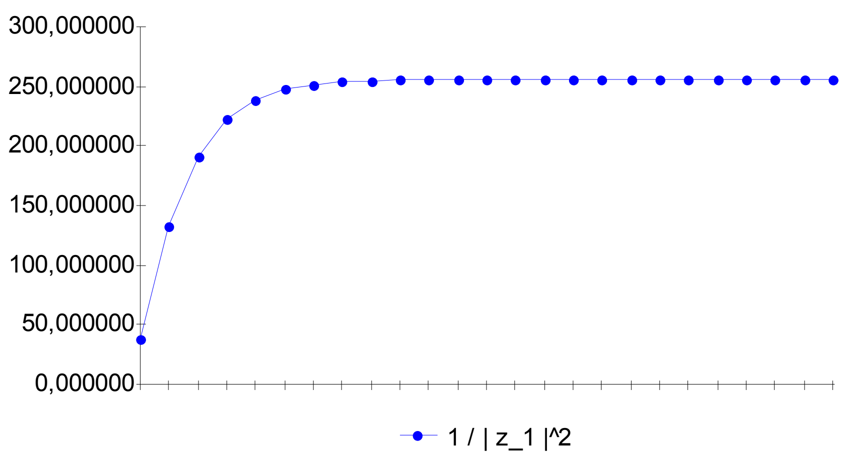
$$\alpha \leq \frac{1}{|z|^2} < \Omega \tag{77}$$

Where alpha is the lowest possible value, the absolute value of *z* is the modulus of *z* and Omega is the upper limit.

Proof of Lemma 6:

The limit Omega for equation (77) can be solved without knowing the reserved value of *M* of *z* one, as the inverse of the square of the absolute value of *z* for all the trivial zeros converges rapidly to the Integer two hundred fifty six. Figure 12 shows how quickly the first of these converges on the Integer two hundred fifty six, as do the remaining so similar as to be nearly identical.

FIGURE 7 The function $1 / |z_1|^2$ for the trivial zeros



In order to determine the lowest possible value for alpha from equation (77), as the highest possible value corresponds with a trivial zero, one can use the first non-trvial zero of the zeta function.

$$s_1\text{ (ntz)} \quad = \tfrac{1}{2} + 14.134725...i \tag{78}$$

Applying the Real part of (78) to equation (3), one gets

$$T_{R1}\text{ (ntz)} \quad = \frac{(2\ (1 \div 2) + 1)^{-1} - 4}{4} \tag{79}$$

In closed form

$$= \frac{-7}{8}$$

And because of proof of lemma 3, the numerator of T_R for the first trivial zero of the zeta function equals one, one can invert T_R in order to solve for the denominator. This gives

$$\delta_1 \text{ (ntz)} = \frac{-8}{7} \quad (80)$$

$$= -1.142857...$$

Then apply delta to equation (60) in order to solve for *z*. One gets

$$z_1 = \frac{-(1 + -1.142...) \pm \sqrt{(1 + -1.142...)^2 + (-1.142... \div 4)}}{2} \quad (81)$$

$$z_1 = \frac{1}{14} \pm 0.257539...i \quad (82)$$

By means of the Pythagorean Theorem, the modulus of *z* can be calculated. One gets

$$|z_1| = \sqrt{0.071428...^2 + 0.257539...^2} \quad (83)$$

In closed form

$$|z_1| = \sqrt{\frac{1}{14}} \quad (84)$$

By means of trigonometric rules, the modulus of *z* for the first non-trivial zero of zeta of *s* can be determined.

$$\cos \mathrm{Am}(z_1) = \frac{0.071428...}{\sqrt{0.071428...^2 + 0.257539...^2}} \tag{85}$$

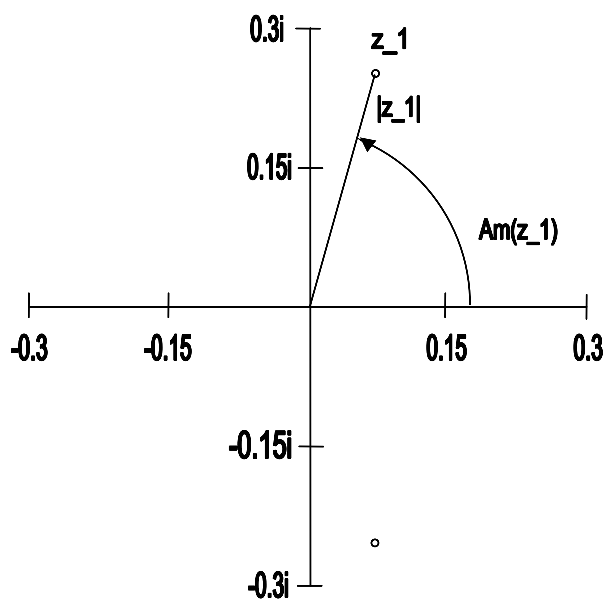
In closed form

$$\cos \mathrm{Am}(z_1) = \frac{1}{\sqrt{14}} \tag{86}$$

Thus,

$$\mathrm{Am}(z) \quad = 1.300246... \tag{87}$$

FIGURE 8 z_1 on the complex plane with its modulus, amplitude and conjugate



By means of the first non-trivial zero of the zeta function being known, one can solve for alpha, the lowest possible value in equation (60). One gets

$$14 \leq \frac{1}{|z|^2} < 256 \tag{88}$$

Where the limits of the modulus squared for all z for both the trivial and non-trivial zeros of the zeta function are defined.

Using equations (73 & 79), the value of H of T_R can be calculated. This gives in closed form

$$H\,(T_{R1}) = \frac{-7}{15} \tag{89}$$

By referring to equation (74) and by means of the rules of a Real number divided by a Complex number, P of T_R can be calculated for the first non-trivial zero of the zeta function. One gets

$$\begin{aligned} P\,(T_{R1}) &= ((1 \div 2)((-7 \div 15) + 1)((-7 \div 8) + 1) \\ &\div ((1 \div 2)^2 + 14.134725...^2)) \\ &+ i\,(-14.134725...\,((-7 \div 15) + 1)((-7 \div 8) + 1) \\ &\div ((1 \div 2)^2 + 14.134725...^2)) \\ &= 0.000166... - 0.004710...i \end{aligned} \tag{90}$$

Then apply P of T_R, of one to equation (58) to solve for K of T_R, of one. This gives

$$\begin{aligned} K\,(T_{R1}) &= \frac{(-7 \div 8) + (-7 \div 8) \bmod (0.000166... - 0.004710...i)}{0.000166... - 0.004710...i} \\ &= -7 - 197.886152...i \end{aligned} \tag{91}$$

Then apply K of T_R, of one to equation (76) to solve back for P of $T_{R,}$ of one. One gets

$$\begin{aligned} P\,(T_{R1}) &= \frac{-1}{-7 - 197.886152...i - 0,5 - 14.134725...i} \\ &= 0.000166... - 0.004710...i \end{aligned} \tag{92}$$

Where equations (90 & 92) are equal.

Repeating the above for all the known non-trivial zeros, one sees that both the Real and Imaginary parts of P of T_R do converge at just the first few zeros, that each modulus for all z equal the square root of one divided by fourteen and all the Real parts of K of T_R equal the Integer minus seven toward infinity.

FIGURE 9 The Real parts of the function P (T_R) for the non-trivial zeros

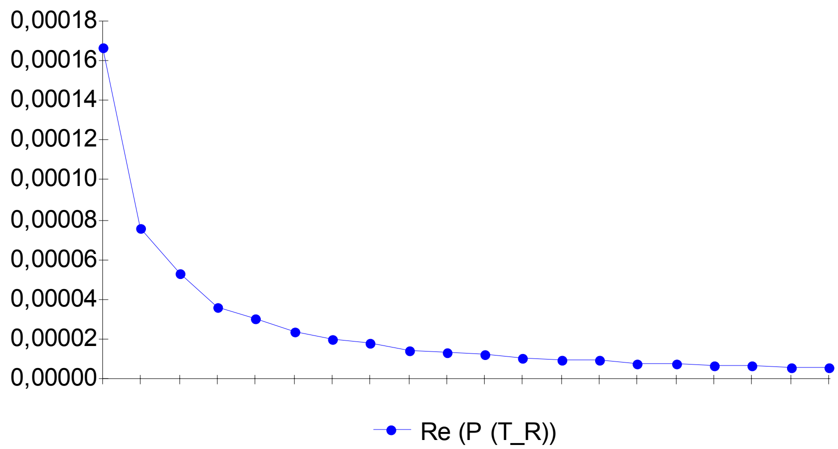
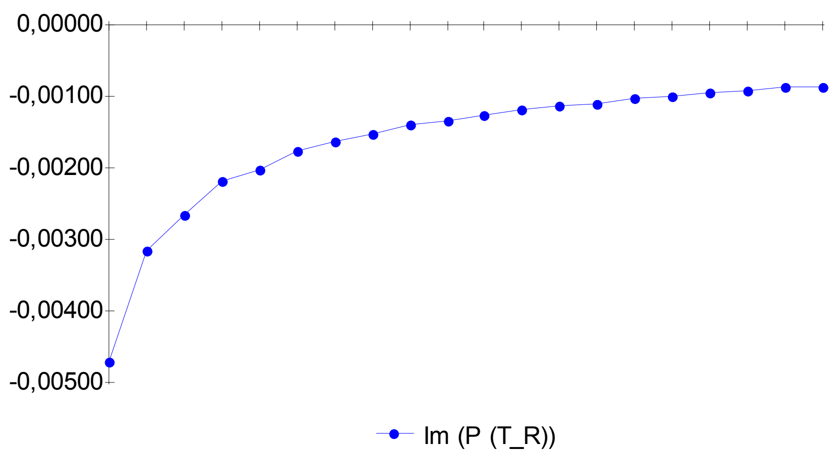


FIGURE 10 The Imaginary parts of the function P (T_R) for the non-trivial zeros



Thus, one can derive an infinite sequence of residues for all the non-trivial zeros

$$\rho_0 = T_{R0} \bmod P\ (T_R) \equiv 1395.783... - 137056.252...i, \tag{93}$$

$$\rho_1 = T_{R1} \bmod P\ (T_R) \equiv 3090.733... - 303161.339...i,$$

$$\rho_2 = T_{R2} \bmod P\ (T_R) \equiv 4376.050... - 429122.495...i\ ...$$

Where P of T_R is a divisor function and T_R is the base in

$$\rho\ (P\ (T_R))_{T\_R} = (\rho_0, \rho_1, \rho_2 ...) \tag{94}$$

Which is the residue sequence for divisor P of T_R, base T_R. Thus, all $\gamma\ (T_R)$ from equations (75 & 95)

$$\gamma\ (T_R) = T_R + T_R \bmod P\ (T_R) \tag{95}$$

Are divisible by $P\ (T_R)$ with no Real remainders, as we see above all K of T_R for the non-trivial zeros have a Real part minus seven.

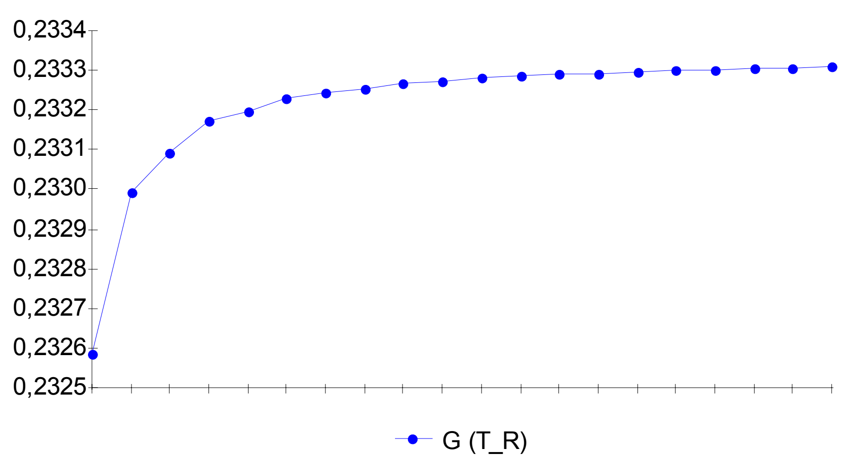
By multiplying all the Real parts of P of T_R by the Real parts of the residues in

$$G\ (T_R) \quad = \mathrm{Re}\ (P\ (T_R)) \times \mathrm{Re}\ (\rho) \tag{96}$$

One gets convergence below in order to demonstrate that all gamma of T_R are divisible by P of T_R without Real remainders based on the lowest possible value of any modulus of z squared for all the non-trivial zeros. One gets

$$\lim_{x \to \infty} G\ (T_R) = \frac{\alpha}{4\ (\alpha + 1)} = 0.233333... \tag{97}$$

FIGURE 11 The Function $G\ (T_R)$ for the non-trivial zeros



And

$$\lim_{x \to \infty} |z| (x) = (\alpha^{-1})^{\lim M\_z (x)} \tag{98}$$

Which allows only two possible amplitudes of *z* for a zero of the zeta function. These are

$$\cos Am(z) = \frac{\pi}{2}, \frac{1}{\sqrt{\alpha}} \tag{99}$$

Such limits restrict all values of T_R of the non-trivial zeros of the zeta function to a single constant. For all T_R of the non-trivial zeros, one gets

$$T_{R1 \to \infty} (ntz) = \frac{-\alpha}{\Omega^{\lim M\_z (x)}} \tag{100}$$

$$= \frac{2 \text{ Re } (K (T_R))}{\Omega^{\lim M\_z (x)}} \tag{101}$$

$$= \frac{2 \text{ H } (T_R) (\alpha + 1)}{\Omega^{\lim M\_z (x)}} \tag{102}$$

$$= \frac{-7}{8} \tag{103}$$

The Riemann Hypothesis:

Given the Riemann Zeta Function in (20), it has been hypothesized by B. Riemann that the Real parts of *s* for all the non-trivial zeros equal one half.

Proof of The Riemann Hypothesis:

By applying the Real part of *s* to the ratio T_R of equation (3) in proof of Lemma one, by means of representing the zeta function by the numerator of T_R in equation (19) in proof of Lemma two, that all nu of the non-trivial zeros equal one in proof of Lemma Three, each modulus of *z* for all the zeros, trivial and non-trivial converge on the square root of one divided by fourteen in proof of Lemma's Four, Five and Six and that T_R for all the non-trivial zeros can be represented as the ratio of the absolute value of the lower limit of the zero's modulus squared divided by the upper limit to the limit of *M* of *z*, of *x*, one can solve for the single common Real part of all the non-trivial zeros, as its Real part is the lowest common denominator of all the Real parts of *P* or T_R in proof of Lemma Six.

From equation (3), apply the value from (103) in order to begin solving for the only Real part possible for a non-trivial zero of the zeta function. This gives

$$\frac{-7}{8} = \frac{(2\mathrm{Re}_{s1\rightarrow\infty}(\mathrm{ntz}) + 1)^{-1} - 4}{4} \tag{104}$$

Multiply both sides by four and then add four. One gets

$$\frac{-28}{8} + 4 = (2\mathrm{Re}_{s1\rightarrow\infty}(\mathrm{ntz}) + 1)^{-1} \tag{105}$$

Reduce the left hand side.

$$\frac{1}{2} = (2\mathrm{Re}_{s1\rightarrow\infty}(\mathrm{ntz}))^{-1} \tag{106}$$

Invert both sides and subtract one from both. This gives

$$1 = 2\mathrm{Re}_{s1\rightarrow\infty}(\mathrm{ntz}) \tag{107}$$

Divide both sides by two, which gives the only possible value of the Real part of the non-trivial zeros of the zeta function.

$$\mathrm{Re}_{s1\rightarrow\infty}(\mathrm{ntz}) = ½ = \lim \mathrm{M}_z(\mathrm{x}) \tag{108}$$

Where the Riemann Hypothesis is true; all the Real parts of *s* for the non-trivial zeros of the zeta function equal one half.